

Quantum Anonymous Veto Protocol

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Abstract: In the classical world, the origin of physical information can, in principle, be traced and hence it seems impossible to setup a secure way for message transmission without revealing its senders' identity. Dining cryptographers (DC) problem introduced by Chaum [1] is one of the primary attempts in this context. In a DC problem, three cryptographers are curious to find out whether their agency NSA (U.S. National Security Agency) or one of them pays for the dinner. At the same time they respect each other's right to make an anonymous payment. A generalized version of the DC problem called DC-net where one of the members from an agency publicizes a secret message without revealing his identity [1]. An unconditionally secure DC-net requires pairwise shared (secure) keys and an authenticated broadcast channel. Since, the security of DC-net relies on the generation of secure key between pairs of members so it is not *unconditionally secure* if members are not allowed to pre-share bilateral private keys. There are other serious flaws in the existing DC protocols e.g., multiple payments, collusion loophole etc.

Anonymous Veto (AV) [2] is also one of the basic primitive for the cryptographic problems like DC problem where the main aim is to hide the identity of the senders of the messages. Here a group of jury members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret *i.e.* without ever disclosing the identity of possible vetoing member(s). This could be very important in many aspects of human societies. Security of the classical solution of this problem is also based either on the *computational hardness* like other classical cryptographic protocols or on imposing restrictions on the number of dishonest players [3]. In this context, Boykin [4] provided a quantum protocol for sending classical information anonymously by distributing pairwise shared *EPR pairs*¹ among players. In 2005, Christandl and Wehner [5] proved that the protocol presented by Boykin is not perfectly secure since it does not satisfy the traceless property and they provided an alternative quantum scheme of the DC-type problem namely 'Quantum Anonymous Transmissions (QAT)' with the traceless feature. In 2007, Brassard *et.al.* presented a more improved QAT protocol [6] and there are some recent works which have also addressed the QAT problem [7]. In this context, following the method proposed by Christandl and Wehner [5] for DC problem, we present a secure quantum protocol for AV problems by employing multi-qubit GHZ correlation [8].

GHZ paradox- In 1989, Greenberger, Horne and Zeilinger (GHZ) [8] provided a way to show a direct contradiction of quantum mechanics with local realism without using any statistical inequality. A simplified version of their argument is as follows: The three qubit *maximally entangled state* $|\Psi\rangle = \frac{|000\rangle - |111\rangle}{\sqrt{2}}$ satisfies following relations

$$\begin{aligned} \sigma_x \otimes \sigma_x \otimes \sigma_x |\Psi\rangle &= (-1) |\Psi\rangle \sigma_x \otimes \sigma_y \otimes \sigma_y |\Psi\rangle = (+1) |\Psi\rangle \\ \sigma_y \otimes \sigma_x \otimes \sigma_y |\Psi\rangle &= (+1) |\Psi\rangle \sigma_y \otimes \sigma_y \otimes \sigma_x |\Psi\rangle = (+1) |\Psi\rangle, \end{aligned} \tag{1}$$

where, $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. Whereas, no local realistic correlation can satisfy above four conditions simultaneously. Hence, no local realistic theory can reproduce quantum correlations.

Quantum Dining Cryptographers (QDC) protocol- In a three-party QDC protocol [5], Alice, Bob and Charlie first share a number (say L_1) of copies of the GHZ state $|\Psi\rangle$, one qubit each from each copy. They randomly selects some of the shared states and run GHZ paradox to check the genuineness of their shared states. The detail of the genuineness check of GHZ state is discussed in the main paper. After confirmation of genuineness of the GHZ states the protocol runs on rest of the copies as follows:

QDC-3: S1. Each member performs σ_z on his qubits if he wants to pay for the dinner otherwise does nothing.

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¹ two-qubit maximally entangled state $\frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]$

- (i) If an even number (*i.e.* zero/two) of members pay the bill (*i.e.*, apply σ_z) then the states of all the members of L_2 remain in the same GHZ state $|\Psi\rangle$ as

$$\begin{aligned} \mathbb{I} \otimes \mathbb{I} \otimes \mathbb{I} |\Psi\rangle &= |\Psi\rangle; \mathbb{I} \otimes \sigma_z \otimes \sigma_z |\Psi\rangle = |\Psi\rangle; \\ \sigma_z \otimes \mathbb{I} \otimes \sigma_z |\Psi\rangle &= |\Psi\rangle; \sigma_z \otimes \sigma_z \otimes \mathbb{I} |\Psi\rangle = |\Psi\rangle. \end{aligned}$$

- (ii) If an odd number (*i.e.* one/three) of members want to pay for the dinner, the states are transformed to $|\Psi^\perp\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$ (which is orthogonal to $|\Psi\rangle$) as

$$\begin{aligned} \sigma_z \otimes \mathbb{I} \otimes \mathbb{I} |\Psi\rangle &= |\Psi^\perp\rangle; \mathbb{I} \otimes \sigma_z \otimes \mathbb{I} |\Psi\rangle = |\Psi^\perp\rangle; \\ \mathbb{I} \otimes \mathbb{I} \otimes \sigma_z |\Psi\rangle &= |\Psi^\perp\rangle; \sigma_z \otimes \sigma_z \otimes \sigma_z |\Psi\rangle = |\Psi^\perp\rangle. \end{aligned}$$

S2. Randomly select a copy of the states to distinguish between the cases (i) even and (ii) odd no. of payment(s). The distinguishability task between case (i) and case (ii) reduces to the problem of distinguishability of two orthogonal three-qubit states $|\Psi\rangle$ and $|\Psi^\perp\rangle$. This can be done by local operation and classical communication [10] and this process in no way, reveals the identity of the operation of the players.

S3. Members randomly selects one of the run and perform the following to distinguish ‘no pay’ vs. ‘double pay’ in case (i) and ‘single pay’ vs. ‘triple pay’ in case (ii). If case (i) occurs, then each member measures σ_y on his qubit if (s)he pays for the dinner otherwise measures σ_x . If the product of the local measurements is -1 then no member has paid for the dinner (*i.e.*, *zero pay*) and if the product is $+1$ then two members have paid for the dinner (*i.e.*, *double pay*). If *zero pay* occurs NSA will pay for the dinner and if *double pay* occurs the payment will be cancelled.

In case (ii), each member measures σ_x on his qubit if (s)he pays for the dinner otherwise, measures σ_y . If the product of the local measurements is $+1$ then all the three members have paid for the dinner (*i.e.*, *triple pay*) and if the product is -1 then only one member has paid for the dinner (*i.e.*, *single pay*). If a *single pay* occurs payment will be accepted otherwise payment will be cancelled.

QDC-n: The generalized version of the QKD protocol with n -party starts with by sharing a large number of copies of generalized n -qubit GHZ state among members

$$|\Psi_n\rangle = \frac{1}{\sqrt{2}} [|000\dots 0\rangle - |111\dots 1\rangle]. \quad (2)$$

The above GHZ state has the following correlation:

$$\begin{aligned} \sigma_z^{\bar{t}} |\Psi_n\rangle &= \frac{1}{\sqrt{2}} [|000\dots 0\rangle - (-1)^t |111\dots 1\rangle] = \frac{1}{\sqrt{2}} [|000\dots 0\rangle - |111\dots 1\rangle] = |\Psi_n\rangle \quad (\text{if } t \text{ is even}), \\ &= \frac{1}{\sqrt{2}} [|000\dots 0\rangle + |111\dots 1\rangle] = |\Psi_n^\perp\rangle \quad (\text{if } t \text{ is odd}), \end{aligned} \quad (3)$$

where $\sigma_z^{\bar{t}}$ denotes the case where σ_z acts on t -number of qubits and no action on rest of the qubits. For the generalized DC problem with no multiple payments $t = 0, 1$. Same kind of protocol with step S1 and S2 as described above will work in this case. By exploiting this generalized QDC protocol we now provide a secure quantum protocol for the AV problem for odd number of parties. Extension the result for even number of parties from odd case is quite trivial.

Quantum Anonymous Veto (QAV) protocol- Imagine a jury with n members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret. The generalized GHZ state $|\Psi_n\rangle$ given in (2) would allow them to achieve this. The quantum AV protocol starts with sharing L ($L \geq 2$) genuinecopies of $|\Psi_n\rangle$ between jury members. Each member gets one qubit from each of the copy of $|\Psi_n\rangle$. Now the QAV protocol with odd number of members goes as follows

S1' Each member performs σ_z on his qubits if he wants to vote ‘against’ otherwise, does nothing.

- (i) If an even number of members (including zero) vote ‘against’ the decision, all the states remain same as $|\Psi_n\rangle$.
- (ii) Otherwise, all the states transform to $|\Psi_n^\perp\rangle$.

S2' Jury members randomly select one copy of the state to distinguish between the cases (i) and (ii) by distinguishing two orthogonal states [10]. Unanimity in favor of the decision happens only if no members (*i.e.* zero members) voted against. Since, (ii) represents the case where at least one of the members voted against so it does not require any farther analysis. But, case (i) represents (a) the unanimity 'in favor' of the decision and (b) an even number (2, 4, ...) of members voted against the decision.

S3' To distinguish between subcases (a) and (b) they first randomly select one copy of the state and each member performs (again) the unitary operation $\sigma_z(1) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{pmatrix}$ on his qubit if he is 'against' the decision, otherwise does nothing.

- (i) If the number (including zero) of members against the decision is even multiple of 2 (*i.e.*, multiple of 2^2) then the selected state will remain in $|\Psi_n\rangle$.
- (ii) Otherwise, (*i.e.*, the number of members against the decision is odd multiple of 2) it will transform to $|\Psi_n^\perp\rangle$.

After distinguishing between these two cases, further analysis has to be made for case (i) in S3'. Case (i) represents (1a) the unanimity in favor of the decision and (1b) multiple of 2^2 (*i.e.*, 4, 8, 12, 16, 20, ...) no. of members voted against the decision.

Jury members keep repeating these steps. In general, to distinguish the case of even multiple of 2^t no. of members (including zero) and the odd multiple of 2^t no. of members against the decision, the required unitary operation will be $\sigma_z(t) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2^t} + i \sin \frac{\pi}{2^t} \end{pmatrix}$. Since, the total number of jury members are finite so after a finite number of steps they can detect whether there is any unanimity 'in favor' of the decision.

If the number of jury members $n(> 2)$ is even then members share copies of $|\Psi_{n+1}\rangle$ where one (say, first) of the jury members holds two qubits from each copy. Except the first, all the other members follow the similar protocol as described above. The first member treats his first qubit as earlier *i.e.*, performs operation/measurement according to his choice of decision and on the second qubit he always performs the operations according to decision 'in favor'. Obviously, this arrangement does not provide any advantage to the first member and hence does not effect the objectivity of the protocol.

The security of our QAV protocol is based on the properties of the correlations of GHZ (2) and its variant (3). The complementary correlation coming from GHZ and its variant are playing a vital role in discriminating various cases that arise in AV. The protocol to check the genuineness of the GHZ state by exploiting the non-local properties of $|\Psi_n\rangle$ are described in main paper. One should note that in contrast to DC problem and other existing quantum anonymous transmissions (QAT) problems [5, 6], the goal of the AV problem is quite different. The existing QAT protocols need prior secure quantum and classical channels and still lack inclusive security proof. On the other hand, in the quantum AV protocol, the security can be guaranteed by sharing quantum resources only.

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Quantum Anonymous Veto Protocol

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Anonymous Veto (AV) is one of the basic primitive for the cryptographic problems like Dining cryptographers (DC) where the main aim is to hide the identity of the senders of the messages. It can be achieved by classical methods where the security is based either on computational hardness or on shared private keys. In this regard, we present a secure quantum protocol for the AV problem by exploiting the GHZ correlations. First, we solve a generalized version of the DC problem with the help of multiparty GHZ state. This allow us to provide a secure quantum protocol for the AV. Security of our protocol rely on some novel and fundamental features of GHZ correlations related to quantum nonlocality.

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I. INTRODUCTION

In the classical world, the origin of physical information can, in principle, be traced and hence it seems impossible to setup a secure way for message transmission without revealing its senders' identity. Dining cryptographers (DC) problem introduced by Chaum [1] is one of the primary attempts in this context. In a DC problem, three cryptographers are curious to find out whether their agency NSA (U.S. National Security Agency) or one of them pays for the dinner. At the same time they respect each other's right to make an anonymous payment. A generalized version of the DC problem called DC-net where one of the members from an agency publicizes a secret message without revealing his identity [1]. An unconditionally secure DC-net requires pairwise shared (secure) keys and an authenticated broadcast channel. Since, the security of DC-net relies on the generation of secure key between pairs of members so it is not *unconditionally secure*¹ if members are not allowed to pre-share bilateral private keys. Another major flaw of the existing DC protocol is that, it can not discriminate among various cases of multiple payments. For example, it shows zero pay *i.e.* no transmission of message if even number (0, 2, ...) of members pay for the dinner and detects payment if an odd (1, 3, ...) number of members pay for the dinner. There is another loophole in the DC protocol namely collusion loophole where some of the participants may cooperate among themselves to trace the person who pays. There are some works that partially resolve the problems related to multiple payments and the collusion problem but none of them provides an unconditionally secure solution [2–4].

There is a variant of DC problem popularly known as *Anonymous Veto (AV)* problem [5]. Here a group of

jury members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret *i.e.* without ever disclosing the identity of possible vetoing member(s). This could be very important in many aspects of human societies. Security of the classical solution of this problem is also based either on the *computational hardness* like other classical cryptographic protocols or on imposing restrictions on the number of dishonest players [6]. In this context, Boykin [7] provided a quantum protocol for sending classical information anonymously by distributing pairwise shared *EPR pairs*² among players. In 2005, Christandl and Wehner [8] proved that the protocol presented by Boykin is not perfectly secure since it does not satisfy the traceless property and they provided an alternative quantum scheme of the DC-type problem namely 'Quantum Anonymous Transmissions (QAT)' with the traceless feature. In 2007, Brassard *et.al.* presented a more improved QAT protocol [9] and there are some recent works which have also addressed the QAT problem [10]. In this context, following the method proposed by Christandl and Wehner [8] for DC problem, we present a secure quantum protocol for AV problems by employing multi-qubit GHZ correlation [11].

We start with a brief description of the GHZ paradox and a quantum protocol for the three-party DC problem based on three qubit GHZ correlation with a detection of multiple payments. We then describe an extension of the protocol to n -party DC problem without any detection of multiple payments. By exploiting this generalized version of DC problem, finally we demonstrate a quantum protocol for the AV problem.

II. GHZ PARADOX

In 1989, Greenberger, Horne and Zeilinger (GHZ) [11] provided a way to show a direct contradiction of quantum

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¹ security based on *computational hardness* of same nature as classical key distribution protocol.² two-qubit maximally entangled state $\frac{1}{\sqrt{2}} [|00\rangle - |11\rangle]$

mechanics with local realism without using any statistical inequality. Consider a three qubit *maximally entangled*³

$$|\Psi\rangle = \frac{|000\rangle - |111\rangle}{\sqrt{2}} \quad (1)$$

known as GHZ states. This GHZ state satisfies the following four relations;

$$\begin{aligned} \sigma_x \otimes \sigma_x \otimes \sigma_x |\Psi\rangle &= (-1)|\Psi\rangle \\ \sigma_x \otimes \sigma_y \otimes \sigma_y |\Psi\rangle &= (+1)|\Psi\rangle \\ \sigma_y \otimes \sigma_x \otimes \sigma_y |\Psi\rangle &= (+1)|\Psi\rangle \\ \sigma_y \otimes \sigma_y \otimes \sigma_x |\Psi\rangle &= (+1)|\Psi\rangle, \end{aligned} \quad (2)$$

where, $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. Then, one can easily show that the above four constraints cannot be satisfied simultaneously by any local-realistic (LR) theory [11, 12]. Similarly, for another three-qubit GHZ state

$$|\Psi^\perp\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}} \quad (3)$$

we have,

$$\begin{aligned} \sigma_x \otimes \sigma_x \otimes \sigma_x |\Psi^\perp\rangle &= (+1)|\Psi^\perp\rangle \\ \sigma_x \otimes \sigma_y \otimes \sigma_y |\Psi^\perp\rangle &= (-1)|\Psi^\perp\rangle \\ \sigma_y \otimes \sigma_x \otimes \sigma_y |\Psi^\perp\rangle &= (-1)|\Psi^\perp\rangle \\ \sigma_y \otimes \sigma_y \otimes \sigma_x |\Psi^\perp\rangle &= (-1)|\Psi^\perp\rangle. \end{aligned} \quad (4)$$

Like the previous case, the above correlation also cannot be satisfied simultaneously by any LR theory.

III. QUANTUM DINING CRYPTOGRAPHERS (QDC) PROTOCOL

First we describe the simplest case of QDC protocol[8]. Imagine that three cryptographers Alice, Bob and Charlie participate in the Dining Cryptographers (DC) problem. They first share a number (say L_1) of copies of the GHZ state $|\Psi\rangle$ given in (1), one qubit each from each copy. Here a copy of the states corresponds to a run of the protocol. Onward we use both the notations *copy of the state* or *run of the protocol* synonymously. After receiving all the qubits from L_1 copies of GHZ states they randomly select some runs (say L_2) and check whether the selected states satisfy the GHZ paradox or not. If yes, rest of the shared states (say, $L = L_1 \setminus L_2$) are genuine copies of GHZ state (1). The detail of the genuineness check of GHZ state is discussed later. After confirmation of genuineness of the states the protocol

runs as follows:

Protocol: QDC(3)
S1. Each member performs σ_z on his qubits if he wants to pay for the dinner otherwise does nothing.
S2. Randomly select a copy of the states to distinguish between the cases (i) even and (ii) odd no. of payment(s).
S3. Distinguish ‘no pay’ vs. ‘double pay’ in case (i) and ‘single pay’ vs. ‘triple pay’ in case (ii).

S1: *Performing local unitary operation to encode payment:* Alice performs local unitary operation σ_z on each of her qubits from L if she wishes to pay the dinner. Otherwise, she does nothing. Bob and Charlie follow the same.

(i): If an even number (*i.e.* zero/two) of members pay the bill (*i.e.*, apply σ_z) then the states of all the members of L_2 remain in the same GHZ state (1) as

$$\begin{aligned} \mathbb{I} \otimes \mathbb{I} \otimes \mathbb{I} |\Psi\rangle &= |\Psi\rangle; \mathbb{I} \otimes \sigma_z \otimes \sigma_z |\Psi\rangle = |\Psi\rangle; \\ \sigma_z \otimes \mathbb{I} \otimes \sigma_z |\Psi\rangle &= |\Psi\rangle; \sigma_z \otimes \sigma_z \otimes \mathbb{I} |\Psi\rangle = |\Psi\rangle. \end{aligned}$$

(ii): If an odd number (*i.e.* one/three) of members want to pay for the dinner, the states are transformed to $|\Psi^\perp\rangle$ given in (3) as

$$\begin{aligned} \sigma_z \otimes \mathbb{I} \otimes \mathbb{I} |\Psi\rangle &= |\Psi^\perp\rangle; \mathbb{I} \otimes \sigma_z \otimes \mathbb{I} |\Psi\rangle = |\Psi^\perp\rangle; \\ \mathbb{I} \otimes \mathbb{I} \otimes \sigma_z |\Psi\rangle &= |\Psi^\perp\rangle; \sigma_z \otimes \sigma_z \otimes \sigma_z |\Psi\rangle = |\Psi^\perp\rangle. \end{aligned}$$

S2: *Distinguishing case (i) and case (ii):* To distinguish between case (i) and case (ii), members randomly select one of the run (say r_1 -th) from L . Now the task is to identify the state ($|\Psi\rangle$ or $|\Psi^\perp\rangle$) corresponding to the run r_1 . Thus, the distinguishability task between case (i) and case (ii) reduces to the problem of distinguishability of two orthogonal three-qubit states $|\Psi\rangle$ and $|\Psi^\perp\rangle$. This can be done by local operation and classical communication [13] and this process in no way, reveals the identity of the operation of the players.

Now the task is to distinguish between subcases ‘zero pay’ vs. ‘double pay’ for case (i) and ‘single pay’ vs. ‘triple pay’ for case (ii).

S3: *Distinguishing between subcases:* Members randomly selects one of the run (say r_2 -th) from $L \setminus \{r_1\}$. If case (i) occurs in the previous step then each member measures σ_y on his qubit if (s)he pays for the dinner otherwise measures σ_x . If the product of the local measurements is -1 then no member has paid for the dinner (*i.e.*, zero pay) and if the product is $+1$ then two members have paid for the dinner (*i.e.*, double pay). The first case follows from the top equation of (2) whereas, the second case follows from the last three equations of (2). If zero pay occurs NSA will pay for the dinner and if double pay occurs the payment will be cancelled.

³ this entanglement is maximal in the sense that it gives the maximum violation of Bell’s inequality for a given set of observables

In case (ii), each member measures σ_x on his qubit if (s)he pays for the dinner otherwise, measures σ_y . If the product of the local measurements is +1 then all the three members have paid for the dinner (*i.e.*, *triple pay*) and if the product is -1 then only one member has paid for the dinner (*i.e.*, *single pay*). The first case follows from the first equation of (4) whereas, the second case follows from the last three equations of (4). If a *single pay* occurs payment will be accepted otherwise payment will be cancelled.

Obviously, two copy of the states, one copy for each step, are sufficient. Therefore, it is enough if the list L contains just two runs. This is true only if all the members honestly follow the entire protocol *i.e.*, perform local unitary operations or measurements (consistently) whenever asked according to their action and declare the true outcomes for each such measurement. If they act dishonestly, they do it solely to trace payer(s) identity only, and not to create any confusion regarding payment. The member who announces his results last in both the steps S2 and S3 enjoys some advantage. (S)He may change the case by just sending a flipped result of her/his measurement outcome. To deal with this problem, members choose more than one run in both steps S2 and S3 and the ordering of the announcement of the result is made random for each such selected run in both steps. Therefore, no members gets the advantage of being last to announce and if someone still flips the result, that will lead to an inconsistent conclusion and subsequently, they abort the protocol and starts a new one with a fresh set of resources.

A. Security analysis of QDC protocol

Since the payer(s) action in step S1 is encoded inside the phase of the GHZ state and the state being party symmetric, no quantum operation can reveal the identity of the payer(s). Step S2 only discloses the information whether the number of payers is odd or even. This information in no way harm the purpose, rather it helps to detect multiple payments. In step S3 members only reveal their individual measurement result and not the choice of measurement to identify the '*no pay*' in case of (i) and a '*single pay*' in case of (ii). By knowing measurement result one cannot predict the measurement choice as that would immediately imply a violation of causality principle. Of course, if two of the members cooperate with each other then they can certainly predict the measurement choice and hence the action of the third party by knowing the measurement result. But this is quite obvious, since the anonymity exists only among a set of possible performers, and if the set is singleton, its member is always traceable *i.e.*, no protocol can keep the singleton members set untraceable. In our QDC protocol if the payment is accepted *i.e.* a '*single pay*' happens then no non-payer have any information about the payer. But, in case of rejection of payment the identity

of payers may be disclosed in two cases (i) if the '*double payment*' occurs then the non-payer knows that the other two members are the payers. This can be avoided if we assume that payment is made by one member only like the original DC problem and then the protocol will end at step S2. Based on the assumption that multiple payments will never occur, one can easily generalize our QDC protocol for $n(\geq 3)$ number of members.

B. Generalized QDC protocol

Let a group of n cryptographers are sitting for dinner at a restaurant and they want to find out whether their agency NSA or one of them pays for the dinner, while respecting each other's right to make a payment anonymously. To implement the protocol n -cryptographers share a copy of the generalized n -qubit GHZ state

$$|\Psi_n\rangle = \frac{1}{\sqrt{2}} [|000\dots 0\rangle - |111\dots 1\rangle]. \quad (5)$$

The above GHZ state has the following correlation:

$$\begin{aligned} \sigma_z^{\bar{t}} |\Psi_n\rangle &= \frac{1}{\sqrt{2}} [|000\dots 0\rangle - (-1)^t |111\dots 1\rangle] \\ &= \frac{1}{\sqrt{2}} [|000\dots 0\rangle - |111\dots 1\rangle] = |\Psi_n\rangle \\ &\hspace{15em} \text{(if } t \text{ is even),} \\ &= \frac{1}{\sqrt{2}} [|000\dots 0\rangle + |111\dots 1\rangle] = |\Psi_n^\perp\rangle \\ &\hspace{15em} \text{(if } t \text{ is odd), } (6) \end{aligned}$$

where $\sigma_z^{\bar{t}}$ denotes the case where σ_z acts on t -number of qubits and no action on rest of the qubits. For the generalized DC problem with no multiple payments $t = 0, 1$. Same kind of protocol with step S1 and S2 as described above will work in this case. By exploiting this generalized QDC protocol we now provide a secure quantum protocol for the AV problem for odd number of parties. Then we extend the result for even number of parties.

IV. QUANTUM ANONYMOUS VETO (QAV) PROTOCOL

Imagine a jury with n members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret. The generalized GHZ state $|\Psi_n\rangle$ given in (5) would allow them to achieve this. The quantum AV protocol starts with sharing L ($L \geq 2$) genuine⁴ copies of $|\Psi_n\rangle$ between jury members. Each member gets one qubit from each of the copy of $|\Psi_n\rangle$.

⁴ To check the genuineness of states they randomly select some copies of them and run the GHZ-type paradox as described in section V.

Protocol: QAV (odd- n)
S1'. Each member performs σ_z on his qubits if he wants to vote 'against' otherwise, does nothing.
S2'. All perform measurement of σ_x on their associated to a selected run to distinguish between the cases (i) even (including zero) and (ii) odd no. of 'against' votes.
S3'. For case (i), distinguish between the cases of (a) unanimity 'in favor' and (b) an even (excluding zero) no. of 'against' votes.

S1': After receiving all the qubits, each member performs the unitary operation σ_z if he is 'against' the decision and does nothing if he is 'in favor'. (i) If an even number of members (including zero) vote 'against' the decision, all the states remain same as $|\Psi_n\rangle$. (ii) Otherwise, all the states transform to $|\Psi_n^\perp\rangle$.

S2': Jury members randomly select one copy of the state to distinguish between the cases (i) and (ii) by distinguishing two orthogonal states [13]. Unanimity in favor of the decision happens only if no members (*i.e.* zero members) voted against. Since, (ii) represents the case where at least one of the members voted against so it does not require any farther analysis. But, case (i) represents (a) the unanimity 'in favor' of the decision and (b) an even number (2, 4, ...) of members voted against the decision.

S3': To distinguish between subcases (a) and (b) they first randomly select one copy of the state and each member performs (again) the unitary operation $\sigma_z(1) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{pmatrix}$ on his qubit if he is 'against' the decision, otherwise does nothing. (i) If the number (including zero) of members against the decision is even multiple of 2 (*i.e.*, multiple of 2^2) then the selected state will remain in $|\Psi_n\rangle$. (ii) Otherwise, (*i.e.*, the number of members against the decision is odd multiple of 2) it will transform to $|\Psi_n^\perp\rangle$.

After distinguishing between these two cases, further analysis has to be made for case (i) in S3.

Case (i) represents (1a) the unanimity in favor of the decision and (1b) multiple of 2^2 (*i.e.*, 4, 8, 12, 16, 20, ...) no. of members voted against the decision.

To distinguish between subcases (1a) and (1b), they again select another copy and each party perform the following unitary operation $\sigma_z(2) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2^2} + i \sin \frac{\pi}{2^2} \end{pmatrix}$ on his respective qubit if he is 'against' the decision, otherwise does nothing. If

an even multiple of 2^2 no. (*i.e.*, multiple of 2^3) of members (including zero) are against the decision the copy remains unchanged. Otherwise, (*i.e.*, an odd multiple of 2^2 no. of members excluding zero are against the decision) the copy transforms zero to $|\Psi_n^\perp\rangle$. Again these two cases can be distinguished by distinguishing the two orthogonal states.

Jury members keep repeating these steps. In general, to distinguish the case of even multiple of 2^t no. of members (including zero) and the odd multiple of 2^t no. of members against the decision, the required unitary operation will be $\sigma_z(t) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2^t} + i \sin \frac{\pi}{2^t} \end{pmatrix}$. Since, the total number of jury members are finite so after a finite number of steps they can detect whether there is any unanimity 'in favor' of the decision.

Note that in the entire protocol the identity of the member giving veto(es) is not revealed. The thing that is revealed is the information regarding the number of vetoes. Here also the security is guaranteed from the genuineness of the GHZ states.

If the number of jury members $n(> 2)$ is even then members share copies of $|\Psi_{n+1}\rangle$ where one (say, first) of the jury members holds two qubits from each copy. Here, except the first jury member all the other members follow the similar protocol as described in case of odd no. of members. In each run, the first member treats the first qubit (from the pair of qubits he received at each run) as earlier *i.e.*, performs operation/measurement according to his choice of decision and on the second qubit he always performs the operations according to decision 'in favor'. Obviously, this arrangement does not provide any advantage to the first member and hence does not effect the objectivity of the protocol.

V. GENUINENESS CHECK OF GHZ STATE

Security of all the protocols described above is solemnly dependent on the genuineness of the corresponding GHZ state. Since, one can construct a secure even $(n-1)$ -parties QAV protocol from an odd n -parties QAV protocol, it is sufficient to describe only the genuineness check of GHZ states for odd n .

Let us consider the following $n+1$ operators

$$\mathcal{O}_0 = \sigma_x^1 \otimes \dots \otimes \sigma_x^{i-1} \otimes \sigma_x^i \otimes \sigma_x^{i+1} \otimes \sigma_x^{i+1} \otimes \dots \otimes \sigma_x^n \text{ and}$$

$$\mathcal{O}_i = \sigma_x^1 \otimes \dots \otimes \sigma_x^{i-1} \otimes \sigma_y^i \otimes \sigma_y^{i+1} \otimes \sigma_x^{i+1} \otimes \dots \otimes \sigma_x^n,$$

for $i = 1, 2, \dots, n$ with the convention $n+1 \equiv 1$. The upper indices on Pauli matrices represent the identity of position. One can easily check that the set of operators $\{\mathcal{O}_i\}_{i=0}^n$ satisfy the following eigenvalue equations

$$\mathcal{O}_i |\Psi_n\rangle = \lambda_i |\Psi_n\rangle, \quad (7)$$

where $\lambda_i = -1$ if $\mathcal{O}_i = \mathcal{O}_0$ otherwise, $\lambda_i = +1$. Now and onwards $\{\mathcal{O}\}_i, i \neq 0$ will imply that the i and $i + i$ -th party measure σ_y and the rest measure σ_x .

To check the genuineness of n (odd)-qubit GHZ state each player randomly selects a run r and correspondingly a t_r from $\{0, 1, 2, \dots, n\}$ and ask all the players to measure their qubit associated with the r -th run in \mathcal{O}_{t_r} and send back their measurement results.

Concerning player collects all the local measurement data (including his won measurement result) associated with the run r and checks whether the product of the local measurement results consistent with the eigenvalue λ_{t_r} of the eigenvalue equation (7). This process will be repeated until all the players are satisfied. Relations given in (7) provide a GHZ like contradiction with LR-theory for an n -qubit system when n is odd. By employing relations given in (7), one can construct the following LR inequality for n -(odd) two level system.

$$\mathcal{O} = \left| \left\langle \sum_{i=1}^n \mathcal{O}_i - \mathcal{O}_0 \right\rangle \right| \leq (n-1). \quad (8)$$

The two extreme eigenvalues of the operator $(\sum_{i=1}^n \mathcal{O}_i - \mathcal{O}_0)$ are $\pm(n+1)$ and the corresponding eigenstates are $|\Psi_n\rangle$ and $|\Psi_n^\perp\rangle$ respectively. Therefore, only for $|\Psi_n\rangle$ and $|\Psi_n^\perp\rangle$, the maximum algebraic value of \mathcal{O} *i.e.*, \mathcal{O} is equal to $n+1$, and hence violates the inequality (8) maximally. Thus, for odd n the relations given in (7) uniquely determines the correlation of $|\Psi_n\rangle$. Therefore, if the product of the local measurement

results associated to the observable \mathcal{O}_{t_r} is equal to the eigenvalue λ_{t_r} , then the correlation is a genuine n -qubit GHZ correlation.

VI. CONCLUSION

In conclusion, we present a secure quantum protocols for the Anonymous Veto (AV) problem. The security of our protocol is based on the properties of the correlations of GHZ (5) and its variant (6). The complementary correlation coming from GHZ and its variant are playing a vital role in discriminating various cases that arise in AV. We also provide a protocol to check the genuineness of the GHZ state by exploiting the nonlocal properties. One should note that in contrast to DC problem and other existing quantum anonymous transmissions (QAT) problems [8, 9], the goal of the AV problem is quite different. The existing QAT protocols need prior secure quantum and classical channels and still lack inclusive security proof. On the other hand, in the quantum AV protocol, the security can be guaranteed by sharing quantum resources only.

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