Experimental detection of steerability for Bell-local states with two measurement settings

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Einstein-Podolsky-Rosen steering [1] lies between entanglement and non-locality in the quantum correlation hierarchy. It is a key resource for one-sided device-independent quantum key distribution (1SDI-QKD) protocols [2] where only one of the two communicating parties needs to trust his or her measurement apparatus to guarantee the information theoretic security of the protocol. With respect to full device-independence which can be derived from non-locality, when both parties do no trust their measurement apparatuses, this greatly reduces the experimental constraints, in particular in the detection efficiencies and propagation losses that can be tolerated. The optimization of 1SDI-QKD protocols for high key rates and minimum implementation requirements is still ongoing. For this, a key ingredient is the choice of a good parameter estimation procedure to ensure the security; one such procedure involves the violation of a steering inequality. A few such inequalities have been proposed so far [3–5] with different requirements in terms of the number of measurement settings and different performance in detecting steerability in mixed states.

Here we present the experimental investigation of a new steering inequality, based on so-called fine-grained inequalities, that was introduced in Ref. [6]:

$$F = \max_{a,b} \left(\frac{1}{2} P(b_P|a_S) + \frac{1}{2} P(b_Q|a_T) \right) \leq F_{\text{LHS}} = \frac{3}{4}, \tag{1}$$

where $P(b_B|a_A)$ is the conditional probability for Bob to get the result b (with $b \in \{+,-\}$) when measuring B (with $B = P, Q$ two maximally incompatible measurement settings), whenever Alice claims that she measured A (with $A = S, T$) and obtained the result a (with $a \in \{+,-\}$). F_{LHS} is the maximum value that Alice could achieve by sending a local hidden state (LHS) to Bob, in a scenario where she does not know P and Q before preparing the state and sending it to Bob.

This inequality requires only two measurement settings (P, Q for Bob, S, T for Alice) and can detect steerable states among generalized bipartite Werner states of the following form:

$$\rho = p \left|\Phi\right\rangle \left\langle\Phi\right| + \frac{1-p}{4} \mathbb{1}, \quad \text{with} \quad \left|\Phi\right\rangle = \sqrt{\alpha} \left|00\right\rangle + \sqrt{1-\alpha} \left|11\right\rangle, \tag{2}$$

where $p, \alpha \in [0;1]$.

If Alice prepares states of the form of Eq. (2) and if the measurement settings are of the form $U = \cos(\theta_U)\sigma_z + \sin(\theta_U)\sigma_x$ (with $U = P, Q, S, T$), with $\theta_Q = \theta_P + \pi/2$ so that P and Q are maximally incompatible, we find that the optimized value of the steering parameter is:

$$\begin{aligned} F = \max_{\theta_S,\theta_T} \Big[\min_{\theta_P} \Big[& \frac{1 + p\big(\sigma|2\alpha-1|\cos(\theta_P) + (\sigma|2\alpha-1| + \cos(\theta_P))\cos(\theta_S) + 2\sqrt{\alpha(1-\alpha)}\sin(\theta_P)\sin(\theta_S)\big)}{4\big(1 + p\big(\sigma|2\alpha-1|\cos(\theta_S)\big)} \\ & + \frac{1 + p\big(-\sigma|2\alpha-1|\sin(\theta_P) + (\sigma|2\alpha-1| - \sin(\theta_P))\cos(\theta_T) + 2\sqrt{\alpha(1-\alpha)}\cos(\theta_P)\sin(\theta_T)\big)}{4\big(1 + p\big(\sigma|2\alpha-1|\cos(\theta_T)\big)} \Big]\Big], \end{aligned} \tag{3}$$

where $\sigma = \text{sign}(\cos(\theta_P + \pi/4))$. Note that Bob needs to find the worst case angle θ_P in order to avoid false positives whenever the state is not symmetric (i.e. $\alpha \neq 1/2$). In Fig. 1a, we plot the theoretical lower bounds for entanglement [7], steering (Eq. (1)) and Bell-nonlocality (CHSH inequality $S \leq 2$ [8, 9]) for generalized Werner states: all states with p larger than these bounds are entangled, steerable or non-local, respectively. Such states violate Eq. (1) for a wide range of p and α, and in particular for Bell-local states, that do not violate Bell's inequalities (region between the red and the green curves). Our steering bound saturates the known bound of $p = 1/2$ for Werner states ($\alpha = 1/2$) and is conjectured to be optimal also for $\alpha \neq 1/2$ as it lies between a lower bound for steerable states and an upper bound for states with a LHS model that were both numerically calculated in Ref. [10].

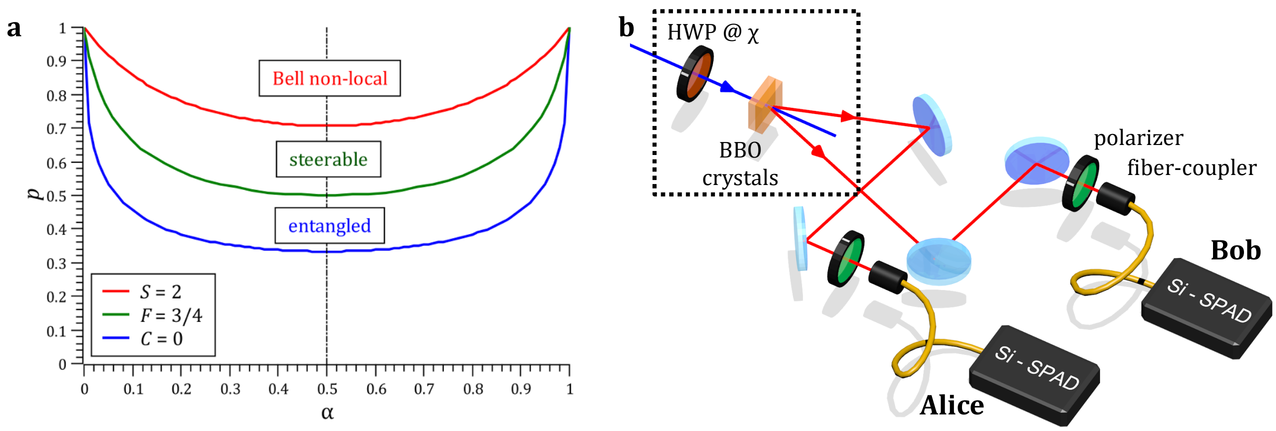

Figure 1. a) Lower p bounds vs α for generalized Werner states. b) Experimental set-up.

We experimentally tested this steering inequality with the set-up shown in Fig. 1b. It consists of a commercial source of polarization-entangled photon pairs ('quED' from QuTools [11]), based on the scheme proposed in Ref. [12]: two thin type-I BBO crystals with their crystal axis orthogonal to each other are pumped by a CW laser beam at 405 nm whose polarization is adjusted by a half-wave plate (HWP) whose axis is oriented with a tunable angle χ with respect to the vertical direction (see the dashed line box in Fig. 1b). Horizontally-(Vertically-)polarized photon pairs at 810 nm are generated in the first (second) crystal by the vertical (horizontal) polarization component of the pump beam. The source thus produces a general Werner state in polarization of the form of Eq. (2), with $\alpha = \cos^2(2\chi)$. The unpolarized background noise (optics fluorescence, ceiling lamps...) set the parameter p to 0.90. After the source, one photon is sent to Alice and the other to Bob. At both stations, projective measurements are done with a rotating polarizer and a silicon single-photon avalanche photodiode (SPAD) after coupling in a single-mode fibre. Coincidence counts between Alice's and Bob's detectors are recorded.

In Fig. 2a, we report the measurement results obtained for the steering parameter F (green diamonds) and the Bell-CHSH parameter S [8, 9] (red dots), as a function of α. The mixed lines correspond to the local hidden variable model (LHV) maximum values that saturate the inequalities. The solid lines are theoretical curves for F and S corresponding to a generalized Werner state with $p = 0.90$, and the dashed lines are theoretical curves taking into account a dephasing noise with a parameter $\eta = 0.96$ coming from a slight distinguishability between the emission modes of the two BBO crystals. In Fig. 2b, F is plotted against S. In both graphs, the green shaded area corresponds to experimentally detected steerable Bell-local states with $F > \frac{3}{4}$ and $S \leq 2$. Errors bars are < 0.035 for α, 0.024 for S and 0.02 for F.

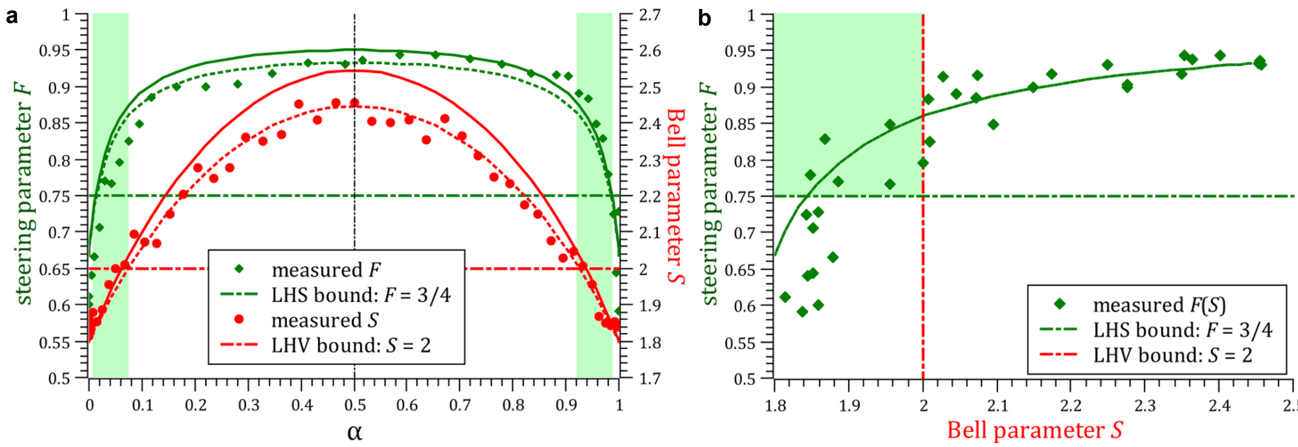

Figure 2. Measurement results and theoretical simulations for generalized Werner states with $p = 0.90$.

Our results show that this fine-grained steering inequality allows to detect steerability with two measurement settings in a wide range of states, in particular in Bell local states where other coarse-grained inequalities require a larger number of measurements settings [4, 5]. In Ref. [6], a secret key rate r for 1SDI-QKD has been derived against individual attacks: $r \geq \log_2[F/(2F_{\text{LHS}} - F]$. However, this rate was proven for a slightly different scenario where Bob's measurement settings are fixed to $P = \sigma_z$ and $Q = \sigma_x$ (and thus known by Alice before state preparation). Thus, in this simpler scenario, the LHS bound is higher ($F_{\text{LHS}} = (1 + 1/\sqrt{2})/2 \approx 0.854$) and steerable generalized Werner states are not all detected. Work is in progress to extend the secret key rate derivation for 1SDI-QKD in the scenario where P and Q are unknown to Alice, a scenario which allows a much better noise tolerance .

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